Name: $\qquad$
Instructor: $\qquad$

## Math 10550, Exam 2

## October 13, 2011

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min .
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

| PLEASE MARK YOUR ANSWERS WITH AN X, not a circle! |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1. (a) | (b) | (c) | (d) | (e) |
| 2. (a) | (b) | (c) | (d) | (e) |
| 3. (a) | (b) | (c) | (d) | (e) |
| 4. (a) | (b) | (c) | (d) | (e) |
| 5. (a) | (b) | (c) | (d) | (e) |
| 6. (a) | (b) | (c) | (d) | (e) |
| 7. (a) | (b) | (c) | (d) | (e) |
| 8. (a) | (b) | (c) | (d) | (e) |
| 9. (a) | (b) | (c) | (d) | (e) |
| 10. (a) | (b) | (c) | (d) | (e) |


| Please do NOT write in this box. |
| ---: | :--- |
| Multiple Choice___ |
| 11. |
| 12. |
| 13. |
| 14. |
| Total |

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## Multiple Choice

1. $(6 \mathrm{pts}$.$) The point P_{0}=(1, \sqrt{2})$ is on the curve whose equation is

$$
\left(y^{2}-1\right)^{3}-x^{2}=0
$$

The equation of the line tangent to the curve at $P_{0}$ is:
(a) $y-\sqrt{2}=\frac{1}{3 \sqrt{2}}(x-1)$
(b) $y+\sqrt{2}=\frac{1}{3 \sqrt{2}}(x-1)$
(c) $y+2=\frac{1}{2 \sqrt{3}}(x-1)$
(d) $y-\sqrt{2}=\frac{-1}{3 \sqrt{2}}(x-1)$
(e) none of the above.
2. ( 6 pts.) Starting at time $t=0$ a particle is oscillating vertically. After t minutes the height of the particle above ground (in feet, upward is positive) is given by

$$
10 \cos (\pi t)
$$

Which one of the statements below is correct when $t=0.25$ minutes? (Only one is)
(a) The particle is below ground, descending and speeding up.
(b) The particle is above ground, descending and slowing down.
(c) The particle is above ground, descending and speeding up.
(d) The particle is below ground, ascending and slowing down.
(e) The particle is above ground, ascending and slowing down.

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3. ( 6 pts .) A police helicopter is hovering in a stationary position 300 ft above a toll gate on an interstate. A car traveling at a constant speed of $100 \mathrm{ft} / \mathrm{sec}$ (That's about 68 mph ) goes through the gate (i-zoom). How fast is the distance between the helicopter and the car increasing when the car is 400 feet from the toll gate?
(a) $70 \mathrm{ft} / \mathrm{sec}$
(b) $65 \mathrm{ft} / \mathrm{sec}$
(c) $60 \mathrm{ft} / \mathrm{sec}$
(d) none of the above.
(e) $80 \mathrm{ft} / \mathrm{sec}$
4. ( 6 pts.) Find the linearization of the function $f(x)=\sqrt[3]{x}$ at $a=125$ and use it to approximate the number $\sqrt[3]{123}$. Which of the following gives the resulting approximation?
(a) $\frac{1}{75}$
(b) $\frac{373}{75}$
(c) $\frac{77}{15}$
(d) $\frac{377}{75}$
(e) $\frac{73}{15}$

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5. ( 6 pts.) Let $f$ be a function which is continuous on the interval $[0,18]$ and differentiable on $(0,18)$. If $f(0)=1$ and

$$
\left|f^{\prime}(x)\right| \leq 2 \quad \text { for all } \quad x \in(0,18)
$$

which statement below must be true? (only one must be, the remaining ones might be false)
(a) $-1 \leq f(4) \leq 3$
(b) $\quad f^{\prime}(4)=2$
(c) $f(x)=1+2 x$
(d) $|f(4)| \leq 2$
(e) $\quad-7 \leq f(4) \leq 9$
6. ( 6 pts .) Which of the following gives a complete list of the critical numbers/points of the function

$$
f(x)=(x+5)^{4}(x-4)^{3} ?
$$

(a) $x=4, \frac{5}{4}$
(b) $x=-5,4$
(c) $\quad x=4, \frac{1}{7}$
(d) $\quad x=-5,4, \frac{1}{7}$
(e) $\quad x=-5,4, \frac{5}{4}$

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7. ( 6 pts.) Let $f(x)=4 x^{5}+5 x^{4}+1$. Which of the following statements is true?
(a) By the first derivative test, $f$ has a local minimum at $x=0$
(b) By the first derivative test, $f$ has a local maximum at $x=0$
(c) By the second derivative test, $f$ has a local maximum at $x=0$
(d) By the second derivative test, $f$ has a local minimum at $x=0$
(e) The nature of the critical point at $x=0$ cannot be determined.
8. $(6$ pts. $)$ Let $f(x)=x^{3}+3 x^{2}-24 x+2011$. Find all local extrema and points of inflection.
(a) $\quad f$ has a local maximum at $x=-4$, a local minimum at $x=-1$ and a point of inflection at $x=2$
(b) $\quad f$ has a point of inflection at $x=-4$, a local minimum at $x=-1$ and a point of inflection at $x=2$
(c) $\quad f$ has a local maximum at $x=-4$, and points of inflection at $x=-1$ and $x=2$
(d) $\quad f$ has a local maximum at $x=-4$, a local minimum at $x=2$ and a point of inflection at $x=-1$
(e) $\quad f$ has a local minimum at $x=-4$, a local maximum at $x=2$ and a point of inflection at $x=-1$

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9. (6 pts.) Let $f(\theta)=\frac{\theta^{2}}{2 \sqrt{2}}+\sin \theta$, where $0 \leq \theta \leq 2 \pi$. On which of the following intervals is the graph of $f$ concave down?
(a) $\left(\pi, \frac{3 \pi}{2}\right)$
(b) $(\pi, 2 \pi)$
(c) $\left(\frac{\pi}{4}, \frac{3 \pi}{4}\right)$
(d) $\left(\frac{3 \pi}{2}, 2 \pi\right)$
(e) $\quad\left(0, \frac{\pi}{4}\right)$

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10. ( 6 pts.) Let $f$ be a function of $x$. The table below shows whether the functions $f^{\prime}(x)$ and $f^{\prime \prime}(x)$ are positive, negative or have value 0 at each of the given values of $x$.

| $x$ | -2 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $f^{\prime}(x)$ | 0 | 0 | 0 |
| $f^{\prime \prime}(x)$ | $>0$ | $<0$ | $>0$ |

Which of the graphs shown below is a feasible graph of $f(x)$ ?
(Note that the label for each graph is given on the lower left of the graph.)

(b)


(c)
(a)
(e) None of the above
(d)


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## Partial Credit

You must show your work on the partial credit problems to receive credit!
11.(10 pts.) Let $f(x)=x^{3}-3 x^{2}+6 x$ on the interval $[0,3]$. Check that the hypotheses of the Mean Value Theorem are satisfied for this function on this interval, and find all numbers $c$ in the interval $(0,3)$ for which

$$
f^{\prime}(c)=\frac{f(3)-f(0)}{3} .
$$

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12.(10 pts.) A box with a square end as shown in the figure below is being deformed by increasing $a$ and decreasing $b$ at a constant rate of $\frac{1}{2}$ inch $/ \mathrm{min}$.


The starting dimensions of the box are $3 \times 2 \times 2$ inches $^{3},(a=3, b=2)$.
(a) When $a=4$, what is the value of $b$ ?
(b) Find $\frac{d V}{d t}$ when $a=4$ inches, where $V$ denotes the volume of the box.

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13.(10 pts.)

The table below shows what is known about a function $f$ which is defined and continuous on the interval $[-1,2]$. The table gives the values of $f, f^{\prime}$ and $f^{\prime \prime}$ at the points given and tells whether $f^{\prime}$ and $f^{\prime \prime}$ are positive or negative on the intervals given.

| $x$ | -1 | $(-1,0)$ | 0 | $(0,0.5)$ | 0.5 | $(0.5,1)$ | 1 | $(1,2)$ | 2 |
| :---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| $f(x)$ | 0 |  | 1 |  | 0 |  | -1 |  | -2 |
| $f^{\prime}(x)$ |  | $>0$ | 0 | $<0$ |  | $<0$ | 0 | $<0$ |  |
| $f^{\prime \prime}(x)$ |  | $<0$ |  | $<0$ | 0 | $>0$ | 0 | $<0$ |  |

Sketch the graph of $y=f(x)$ using all of the above data on the axes provided.


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14. (10 pts.) Find the absolute minimum of the function

$$
f(x)=x^{2 / 3}(x-2)^{2}
$$

on the interval $[-1,1]$.

Absolute minimum at $x_{0}=$ $\qquad$ , $f\left(x_{0}\right)=$ $\qquad$ .

